

ANALYSIS OF WIND POWER FOR BATTERY CHARGING

Eduard Muljadi

Stephen Drouilhet

Richard Holz

Wind Technology Division
National Renewable Energy Laboratory
Golden, Colorado

Vahan Gevorgian

State Engineering University of Armenia
Yerevan, Armenia

ABSTRACT

One type of wind-powered battery charging will be explored in this paper. It consists of a wind turbine driving a permanent magnet alternator and operates at variable speed. The alternator is connected to a battery bank via rectifier. The characteristic of the system depends on the wind turbine, the alternator, and the system configuration. If the electrical load does not match the wind turbine, the performance of the system will be degraded. By matching the electrical load to the wind turbine, the system can be improved significantly.

This paper analyzes the property of the system components. The effect of parameter variation and the system configuration on the system performance are investigated. Two basic methods of shaping the torque-speed characteristic of the generator are presented. The uncompensated as well as the compensated systems will be discussed. Control strategies to improve the system performance will be explored. Finally a summary of the paper will be presented in the last section.

I. INTRODUCTION

Electrical energy produced by a wind turbine has many uses. Converting wind energy into electric energy enables the user to store energy in a battery, transmit it over long distances, or convert the energy into many different forms (mechanical energy, heat, etc.). Most of the large wind turbines are connected to the grid. In some small applications, wind turbines are operated in isolated operation [1]. Battery charging is very popular because of its simplicity and versatility. DC or AC generators can be used. While many papers on battery charging have been written [2,3], a qualitative analysis needs to be presented.

The major goal of this paper is to model and analyze the system, and to predict the performance improvements that can be achieved by altering the system configuration to better match the load to the wind turbine.

By understanding the basic characteristics of the components, many of the performance limitations of the system can be remedied and the optimization of the system can be explored. The approach considered is to insert a load controller between the generator and the load. Different types of controllers will be investigated, including a power converter and a capacitor-compensated system.

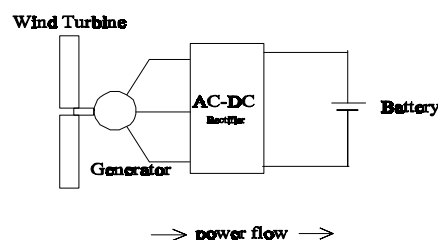


FIGURE 1. PHYSICAL DIAGRAM OF THE SYSTEM

The physical diagram of the system under investigation can be shown in Figure 1. The three-phase alternator is connected to a three-phase rectifier, the output of which is connected to a battery bank. In the conventional system, there is no active control used to adjust the energy produced by the wind turbine; therefore, the power flow to battery is dictated solely by the wind speed and the passive interaction of the various system components.

This paper is organized as follows: In Section II, the wind turbine aerodynamic characteristics are discussed. The equations describing the alternator will be presented in Section III. In Section IV, the compensated system is presented. Both the DC converter and the series capacitor compensation system will be presented. Finally, the conclusion will be presented in

Section V.

II. Wind Turbine Aerodynamic Characteristics

The wind turbine is normally characterized by its C_p -TSR curve, here TSR is the tip-speed ratio, the ratio of the linear speed of the blade tip to the wind speed. A typical C_p -TSR curve is given in Figure 2. It is desirable that the wind turbine is operated at high C_p values most of the time.

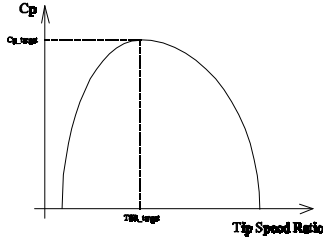


FIGURE 2. TYPICAL CP-TSR CHARACTERISTIC OF A WIND TURBINE

In a fixed-frequency application, typical of larger wind farm turbines, the rotor speed of the generator is fixed, while the speed of the wind may vary in a wide range. Thus the tip-speed ratio may vary over a wide range. The power captured by the wind turbine rotor may be written as

$$P_{\text{mech}} = 0.5 \cdot A \cdot C_p \cdot V^3 \quad (1)$$

where:

ρ air density kg/m^3
 A swept area m^2
 C_p coefficient of wind turbine
 V wind velocity m/sec

From the equation, it is apparent that the power production from the wind turbine can be maximized if the system is operated at $C_{p\text{-target}}$. Thus, it is necessary to keep the rotor speed at constant TSR (i.e., at $\text{TSR}_{\text{target}}$).

As the wind speed changes, the rotor speed should be adjusted to follow the change. Unfortunately, wind speed at the rotor is difficult to measure. To avoid using the wind speed, the equation to compute the target power can be revised. By substituting the wind speed V and the C_p , the target power P_{target} can be derived [4].

As an illustration, the mechanical power generated by the wind turbine as a function of rpm for different wind speed is shown in Figure 3. The target power as a function of rpm is shown on the same graph. It is clear that for any wind speed, there is always a

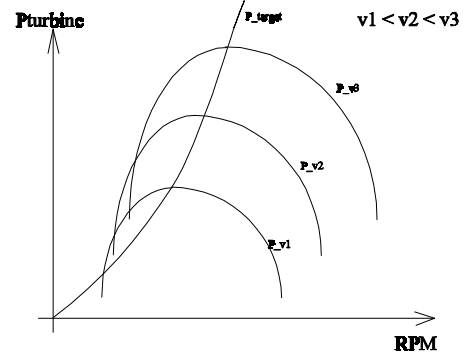


FIGURE 3. POWER AT DIFFERENT WIND SPEED

matching rotor speed which produces maximum power. If the controller can successfully follow the changes in wind speed, the wind turbine will generate maximum power at any speed. The target torque of the wind turbine can be derived from the target power and it can be written as follows:

$$T_{\text{target}} = \frac{K_W \cdot P_{\text{target}}}{\omega^2} \quad (2)$$

where:

K_W target $0.5 \cdot A \cdot C_{p\text{-target}} \left[\frac{R}{\text{TSR}_{\text{target}}} \right]^3$
 $C_{p\text{-target}}$ target power coefficient
 $\text{TSR}_{\text{target}}$ target tip speed ratio
 R radius of the blade m
 ω rotor speed mechanical rad/s

III. Uncompensated Generator Characteristics

The generator considered is a three-phase permanent magnet alternator. The generator is connected to a DC battery bank via a rectifier. With a diode bridge rectifier, the effective load across the generator terminal is a resistive load whose resistance is varied to get a constant DC voltage. Thus a unity-power-factor load condition is a constraint that must be maintained across the generator terminal. The per-phase equivalent circuit representing the system is shown in Figure 4. The rectifier-battery unit is presented as an AC voltage source with a symbol of DC and AC source with a small arrow inside the circle to signify the constraint of constant voltage, unity-power-factor, and unidirectional power flow.

In the steady state-analysis, the rectifier and the battery are presented to be a purely sinusoidal source with a unity power factor current flowing into the load. It is assumed that the generator consists of only the synchronous reactance X_s (the stator resistance R_s is

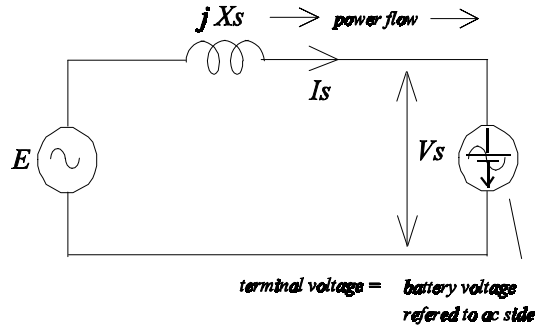


FIGURE 4. SIMPLIFIED PER-PHASE EQUIVALENT CIRCUIT OF THE BATTERY CHARGING SYSTEM

neglected) and the three-phase generator is wye-connected. More detailed discussions of generators and power converters can be found in many electric machine and power electronics textbooks [5-6].

The terminal voltage of the generator is determined by the battery voltage across the DC bus. The relationship between the terminal voltage of the generator and the DC bus voltage can be derived from the balance of power:

$$3 V_s I_s = V_{dc} I_{dc} \quad (3)$$

where:
 V_s per phase stator terminal voltage (AC)
 I_s stator current (AC)
 V_{dc} DC Bus voltage
 I_{dc} DC current

It is easy to verify that the battery voltage can be referred to the AC side. The terminal voltage V_s seen by the generator can be derived by using the concept of power balance. From conservation of energy, the power on the AC side must be equal to the power on the DC side. Assuming that the rectifier operates only in continuous conduction mode, the power balance yields

$$I_s = \frac{V_{dc}}{3V_s} I_{dc} \quad (4)$$

The rectifier converts the three-phase AC voltage into DC voltage. The relationship between V_{dc} and the line to line voltage V_{LL} can be illustrated in Figure 5. The average value of the DC bus voltage can be found by integrating the voltage waveform shown in Figure 5.

$$V_{dc, ave} = \frac{1}{\text{Period}} \int_0^{\text{Period}} v_{dc}(\theta) d\theta \quad (5)$$

$$= \frac{3}{\pi} \int_0^{\pi/6} \sqrt{6} V_{LL, peak} \cos \theta d\theta$$

$$= \frac{3}{\pi} V_{LL, peak}$$

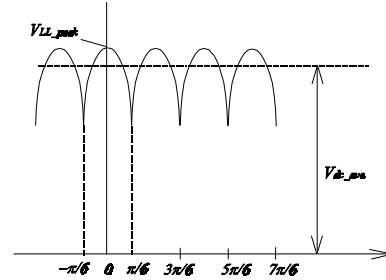


FIGURE 5. OUTPUT OF A RECTIFIER

Expressed in rms, the following equation can be derived.

$$\frac{V_{dc}}{V_{LL}} = \frac{3\sqrt{2}}{\pi} \quad (6)$$

The relationship between V_{dc} and the phase voltage can be written as follows

$$\frac{V_s}{V_{dc}} = \frac{\pi}{3\sqrt{6}} \quad (7)$$

Thus the relationship of the AC stator current and the DC bus current can be found.

$$I_s = \frac{\sqrt{6}}{\pi} I_{dc} \quad (8)$$

The generated emf E voltage of a permanent magnet alternator can be expressed as a function of the rotor speed and the magnetic flux of the airgap. Thus the voltage E is directly proportional to the frequency or rotor speed.

$$E = k_E \omega \quad (9)$$

where:
 k_E constant derived the alternator air gap flux
 ω frequency in rad/s
 f_o output frequency in Hz

The relationship between stator voltage and current can be written as follows:

$$V_s = E - jX_s I_s \quad (10)$$

where
 $X_s = \omega_e L_s$
 L_s synchronous inductance

The phasor diagram in Figure 6 shows that the terminal voltage V_s and the stator current I_s are in phase because of the rectifier. It is easy to verify that the size of stator current is affected by the speed of the generator, the size of the synchronous inductance, and the stator load.

It can be seen that there is a single solution at any frequency. Thus, at any frequency there will be a unique set of stator current, power and power angle. With constant stator terminal voltage, the only way to change the generated torque is to change the frequency.

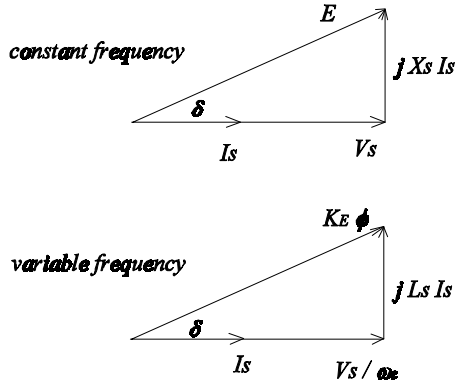


FIGURE 6. PHASOR DIAGRAMS OF AN UNCOMPENSATED SYSTEM.

The equation governing the system can be modified as follows:

$$\frac{V_s}{\omega_e} = K_E - jL_s I_s \quad (11)$$

Thus there is a direct relationship between the terminal voltage and the stator current based on the geometry of the unity power factor concept. The equation to represent the power generated by the alternator and the torque equation can be written as follows:

$$P = 3 \frac{E V_s}{X_s} \sin \delta \quad (12)$$

$$T = \frac{3 P}{2 \pi} \frac{E V_s}{X_s} \sin \delta$$

where
 p number of poles
 δ power angle

The torque equation can be simplified further:

$$T = T_{max} \sin 2\delta \quad (13)$$

where
 $T_{max} = \frac{3 p K_E^2}{4 L_s}$

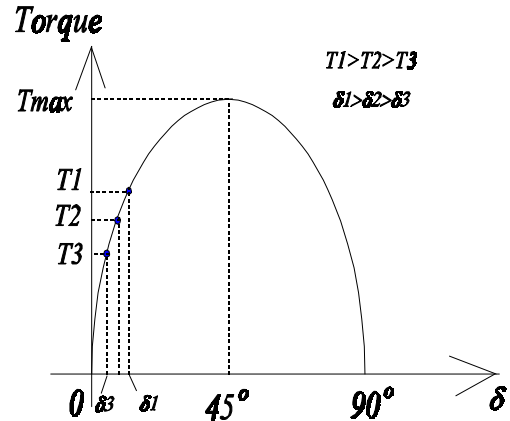


FIGURE 7. TORQUE CHARACTERISTIC OF THE GENERATOR.

It is interesting to note that the maximum torque generated is independent of the frequency. And the power angle (δ) at maximum torque for the system is 45° . In comparison, the power angle at maximum torque of a conventional synchronous generator connected to a utility is 90° . The power angle δ can be found as a function of the frequency or rotor speed.

$$\cos^{-1} \left(\frac{V_s}{K_E} \right) \quad (14)$$

With a constant terminal voltage V_s , the V_s/ω_e at the terminal of the generator varies. The system starts generating when the $V_s/\omega_e < K_E \phi$ ($\delta > 0$). Thus for any particular DC bus voltage, there is a minimum frequency below which the system does not generate. As the rotor speed changes, the power angle δ also changes and the torque follows the curve shown in Figure 7. The stator current can be found by solving the phasor diagram shown in Figure 6.

$$I_s = \frac{K_E}{L_s} \sqrt{1 - \left(\frac{V_s}{K_E} \right)^2} \quad (15)$$

As the frequency increases, the power angle and the stator current I_s increases. Expressed in different way, the generator power can be written as follows:

$$P = \frac{3V_s K_E}{L_s} \sqrt{1 - \left(\frac{V_s}{K_E} \right)^2} \quad (16)$$

It can be observed from these equations that the corresponding stator current and the power generated never reaches maximum, and power angle never reaches 90° (unless the frequency goes to infinity). The power

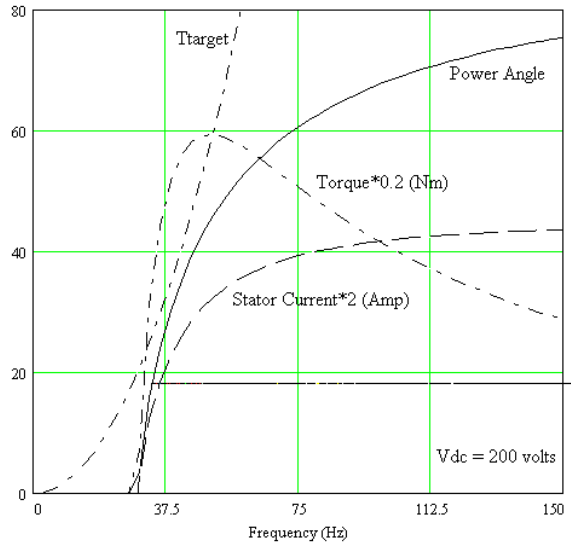


FIGURE 8. TORQUE, CURRENT, AND POWER ANGLE AS FREQUENCY VARIES

angle, the torque, and the stator current as a function of the frequency (or the rotor speed) can be drawn in Figure 8. The torque and stator current have been scaled to fit into the same axis as the power angle. On the same figure, a typical target torque is drawn. In reality the mechanical target torque of the wind turbine has to be limited for mechanical or electrical reasons or both.

The major drawback of the uncompensated mode is that the loading characteristic of the generator cannot be changed, it is determined by the characteristic of the generator (ϕ and L_s) and the DC bus (battery

voltage). Thus as the rotor speed increases, there will be a corresponding, stator current, power, torque and power angle. As shown in Figure 8, the system starts to operate at about 40 Hz when the generator begins to charge the battery. The stator current, power angle increases, the torque reaches its peak at power angle about 45° . As predicted from the equation, the stator current never reach the maximum, and the power angle never reach 90° as the rotor speed increases. The DC bus voltage determines the frequency at which the generator begin to charge the battery. The lower the DC bus voltage, the lower the frequency of the generator before it starts to charge the battery.

IV. Compensated System Characteristics

In an uncompensated system, the generator torque depends on the flux, the synchronous inductance, and the power angle. Thus the torque capability is limited by the characteristic of the synchronous machine. For a constant terminal voltage V_s , the only way to change the torque angle is to adjust the rpm of the generator. Thus without compensation, the characteristic of the wind generation is inherent to the wind turbine and generator. On the other hand, the wind turbine operation can be optimized by changing the characteristic of the load applied to the generator.

To change the load of the generator, the V_s/ω_e at the terminal of the generator should be controlled. The power angle, the power, and the stator current will follow accordingly.

Figure 9 shows a family of torque curves as a function of the output frequency. For single DC bus voltage, the torque varies with frequency as the rotor speed changes. It can be seen that none of the curves fit the target torque which means that at constant DC bus, the wind turbine is not operated in the maximum C_p except at the crossing points between the generator torque curve and the target torque curve.

DC-DC Power Converter Approach

The terminal V_s/ω_e can be controlled by adjusting the DC bus voltage. Referring to Figure 9, the characteristic of the generator can be changed if the terminal voltage V_s is adjustable. To follow the target power, the terminal voltage V_s (i.e., DC bus voltage) must be changed as the rotor speed changes. For one DC bus voltage setting, there can be two, one or no crossing points between the generator power and the target power. As an example, for a DC bus voltage of 150 volts, there are two operating points (point B and point b) that satisfy the target power. Point B corresponds to a lower power

operation at low frequency, lower δ angle, higher terminal V_s/ω_e , and lower stator current. Point b correspond to higher power operation at higher frequency, higher δ angle, lower terminal V_s/ω_e , and higher stator current I_s .

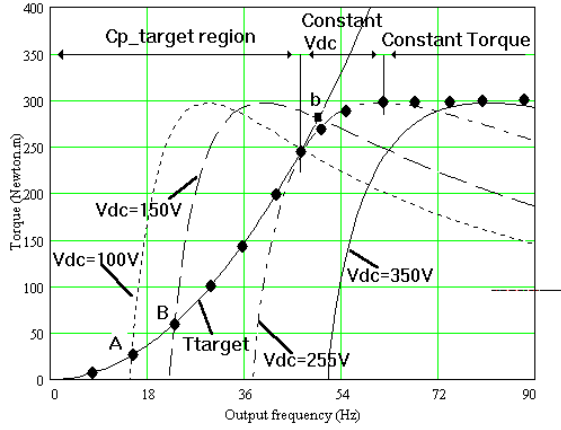


FIGURE 9. A FAMILY OF TORQUE CURVES FOR DIFFERENT DC BUS VOLTAGE SETTINGS

The strategy discussed below is one among many different control algorithms that can be implemented. The controller can be regulated so that the generator operation will be in the lower δ angle as the stator terminal voltage varies. With low δ angle, the generator is always operated at minimum stator current to achieve the same output torque. Thus the copper losses in the stator winding can be minimized. Obviously, as the frequency increases, the power angle increases until a maximum torque is reached at about 45° power angle. From then on, it will be best to keep the torque at its maximum value as frequency increases. The DC-DC converter (DC converter) can be inserted between the rectifier output and the DC battery. It can be used to change the apparent DC bus voltage seen by the generator. Thus by controlling the DC converter the terminal voltage V_s of the synchronous generator is adjustable and V_s/ω_e can be controlled independent of frequency. Figure 9 shows the generator torque with the DC bus set to different values. The operating points of the system control identified by a path of diamond string. Three regions of interest can be distinguished: constant C_{p_target} region, constant DC voltage region, and constant torque (peak torque region). In the lower power region, the generator torque can follow the target torque where the power coefficient can be kept at C_{p_target} . Thus the DC bus voltage is controlled to track the target torque. As the rotor speed increases, the generator torque gets closer to its peak value until it's tangent to the target torque.

From this point, the DC bus voltage is kept constant until the peak torque is reached. This region is called the constant V_{dc} region. As the frequency increases, the torque is kept constant at its peak value and this region can be called the constant torque region. This region is also related to constant V_s/ω_e and constant δ angle (45°).

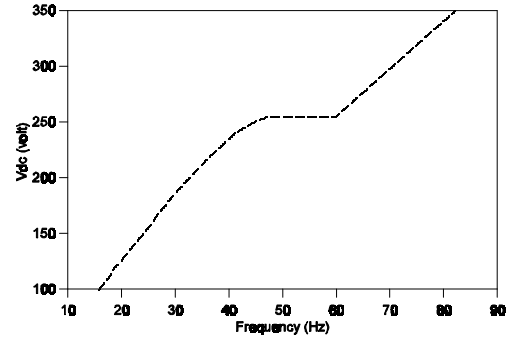


FIGURE 10. CONTROL OF DC BUS VOLTAGE TO GENERATE THE DESIRED TARGET TORQUE

Figure 10 shows how the DC bus voltage can be controlled to adjust the generator torque to follow the path of T_{target} specified along the diamond trace. The values of DC bus voltage versus frequency can be computed given the parameters of the generator and the wind turbine. The values of V_{dc} are controlled based on the frequency or rpm. In the constant C_{p_target} region, the DC bus voltage is varied non linearly and then it is kept constant in the constant DC bus region and finally it is varied linearly in the constant torque region.

Although there are many ways to implement the concept described above, one possible way is to control the duty cycle of the DC converter related to the rpm measured. This can be done in a feed forward fashion which works fine if the DC battery is approximately constant. It can also be done by providing the DC bus voltage feedback. The deviation (error) between the commanded DC bus voltage and the actual DC bus voltage can be used to control the duty cycle of the DC converter.

Another way to control the system is to include a power measurement into the DC side and measure the average power charging the battery. The rpm measured can be used to compute the approximate target torque of the wind turbine (using equation 2). The torque can be measured indirectly, i.e., the measured power is divided by the measured rpm. Any error between computed target torque and measured torque can be used to adjust the duty cycle of the power converter.

The per-phase equivalent circuit representing the compensated system is similar to the one shown in Figure 4 with the exception that the terminal voltage (V_s)

in a compensated system can be changed. The constraint of unity power factor, and unidirectional power flow still exist. Unlike the uncompensated system, the torque can be adjusted by controlling the terminal voltage V_s independent of frequency. It can be shown that reducing the terminal voltage V_s at one particular rpm increases the generator torque until the power angle 45° is reached.

Series Capacitor Compensation

Another method to control the terminal voltage of the synchronous generator is to separate the DC bus from the terminal voltage of the synchronous generator. By separating the terminal voltage from the DC bus, the terminal voltage does not have to be clamped to the DC bus voltage. One possibility is to use a series capacitor inserted between the terminal of the generator and the input to the rectifier.

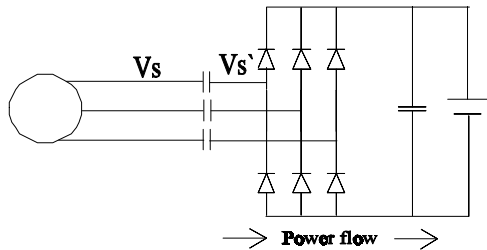


FIGURE 11. PHYSICAL DIAGRAM OF THE SERIES CAPACITOR COMPENSATION

The physical diagram describing the series capacitor compensation can be shown in Figure 11 and the equivalent circuit can be shown in Figure 12. The presence of the capacitor in series with the generator does not change the constraint imposed by the rectifier and battery voltage which is represented by voltage V_s' in the equivalent circuit. The rectifier-battery unit is represented by unity power factor and unidirectional power flow.

There are two different ways of looking at the system. One is to treat the synchronous reactance and the capacitor as a unit impedance connecting two bus voltages. The other one is to treat the generator connected to a unity power factor load in series with the capacitor. As a unit impedance, the total impedance of the load seen by the EMF of the generator is smaller. Thus the total impedance will be $X_s - X_c$ and the torque equation can be derived.

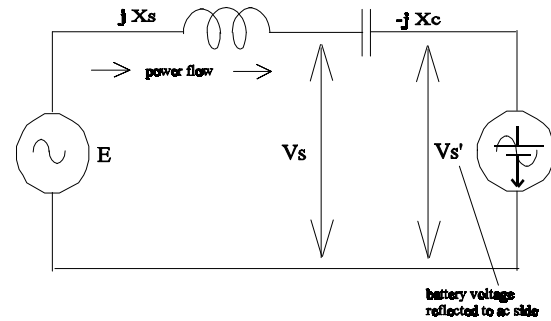


FIGURE 12. PER-PHASE EQUIVALENT CIRCUIT FOR SERIES COMPENSATED SYSTEM.

The torque capability of the system changes as the series capacitor is added to the system. If the synchronous reactance and the series capacitor is treated as a unit, the following equation can be written:

$$T = \frac{3P}{2} \frac{E V_s}{(X_s - X_c)} \sin \delta \quad (17)$$

$$P = 3 \frac{E V_s}{(X_s - X_c)} \sin \delta \quad (18)$$

where

V_s voltage at the rectifier input

$X_s = \omega L_s$

$X_c = \frac{1}{\omega C}$

The torque and power as a function of angle δ' is shown to have adjustable peak values. At any rpm, the peak value of the torque and the power are boosted up by the reactance X_c created by the capacitor which has the opposite sign of the reactance X_s . Thus the overall denominator is smaller.

the voltage across the rectifier V_s' , while the angle δ' is the power angle between the emf E and the terminal voltage V_s .

It is clear that the peak torque varies with frequency as the level of capacitor compensation varies with frequency. The phasor diagram comparing the uncompensated system with the compensated system can be drawn in Figure 13. Note that the angle δ' in the compensated system is an angle between the emf E and the voltage across the rectifier V_s' , while the angle δ is the power angle between the emf E and the terminal voltage V_s .

The compensated system can also be treated as a generator connected to a leading power factor load which consists of a rectifier in series with three-phase

capacitor. The terminal voltage V_s is not constant, and the load current I_s is not in phase with the terminal voltage V_s . It can be expected that the terminal voltage is higher than the voltage across the rectifier input. It is shown in Figure 13 that the installation of the capacitor cancels some of the voltage drop across the synchronous reactance. As a result the voltage across the terminal V_s is higher in the compensated circuit than the uncompensated one.

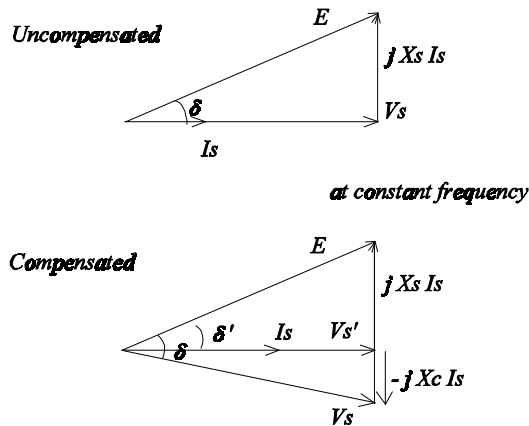


FIGURE 13. PHASOR DIAGRAM FOR SERIES CAPACITOR COMPENSATION

It is shown in the phasor diagram that the compensated system generates a higher stator current and a higher terminal voltage V_s . Thus the peak torque of the compensated system is higher than the uncompensated system. The torque angle δ of the compensated system also increases, creating an overall increase on the output power generated.

Typical current and voltage waveforms of the series compensated system can be shown in Figure 14. It is shown that the terminal voltage V_s is not sinusoidal due to the rectifier connected at the terminal. The current waveform is close to sinusoidal which will have a minimal impact on the torque pulsation on the rotor shaft. Between the terminal of the generator and the rectifier there is a series capacitor installed. The AC voltage across the rectifier input is mainly affected by the DC bus voltage.

The target power from the wind turbine and the power output of the generator as a function of rpm are presented in Figure 15. Three different compensations are shown ($V_{dc} = 180$ volt with series capacitor = $65 \mu F$, $V_{dc} = 180$ volt with series capacitor $C = 220 \mu F$, and $V_{dc} = 350$ volt with series capacitor $C = 220 \mu F$). It is shown that the effect of the lower DC bus enables the generator to operate at lower rpm. The two different sizes of capacitor at the same DC bus voltage have different torque speed characteristics. In the lower speed region, the lower DC bus voltage is closer to the target power,

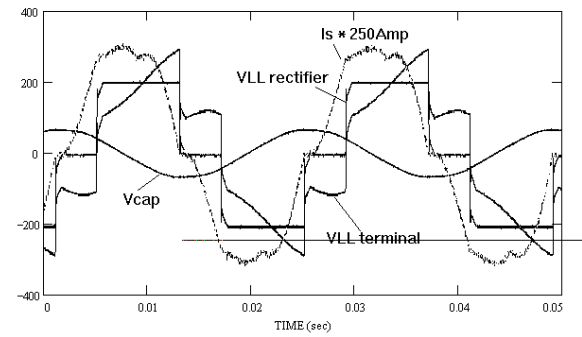


FIGURE 14. VOLTAGE AND CURRENT WAVEFORMS FROM EXPERIMENTAL DATA

and as the rotor speed increases, the DC bus voltage needs to be increased. Thus there is no ideal solution for the series capacitor compensation. However, the torque speed characteristic of the generator can be made to approximate the target power by proper choice of capacitor and DC bus voltage.

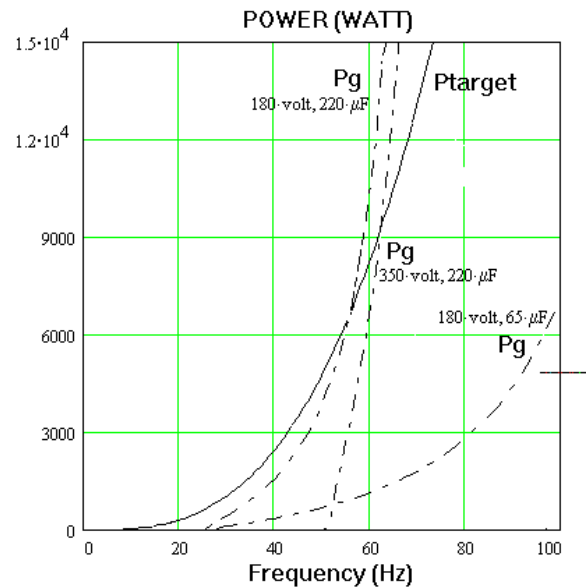


FIGURE 15. GENERATED POWER FOR DIFFERENT SIZES OF CAPACITOR AND DIFFERENT SETTINGS OF DC BUS VOLTAGE

V. CONCLUSION

Our investigation of this battery charging system has shown that the following can be done to improve the energy capture of the wind turbine:

- The characteristics of an uncompensated system are determined by the parameters of the generator. The

torque-speed characteristic of the generator cannot be changed without any active control.

- Compensation with DC-DC converter

- * The wind turbine can be operated at maximum C_p (C_{p_target}) at any rpm (within the range of allowable operation).

- * The torque ($T-\delta$) characteristic has the same peak as the uncompensated generator. However, the torque can be controlled independent of the rotor speed by controlling the terminal voltage V_s (duty cycle of DC converter). Once the peak torque is reached, the DC converter can be used to keep constant torque (at the peak value).

- * As in the uncompensated system, the load presented to the generator is a unity power factor load.

- * There is a need to control the duty cycle of the DC converter to control the terminal voltage V_s at any rpm.

- Compensation with a series capacitor

- * The series compensation can increase the torque/power capability of the generator.

- * The power factor of the load presented to the generator is no longer a unity power factor load. The series capacitor presents a leading power factor to the generator.

- * For a preset value of series capacitor, the torque, power, and current are dependent on the rotor speed and cannot be controlled.

- * There is no control necessary; the torque speed characteristic can approximate the target power characteristic if the capacitor is correctly chosen.

- The next logical step to improve the compensation is to combine the series capacitor compensation and the DC converter approach. In a combined compensation (series capacitor and DC-DC converter) the following characteristics can be expected:

- * With series capacitor the torque capability of the generator is increased; thus operation at maximum C_p can be extended to a wider range.

- * With DC converter, the operation at maximum power capture (C_{p_target}) is possible at any rpm as long as the other operating limit (speed, voltage, and stator current) are not exceeded.

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